Correspondence

Distinctions between Gyroelectric and Gyromagnetic Media in Rectangular Waveguide

Gas plasmas and ferrites become anisotropic when placed in a magnetostatic field. Waveguides containing these media may be termed, respectively, as gyroelectric and gyromagnetic [1]. In this correspondence we compare these waveguides when the magneto-static field is transverse to the direction of wave propagation. In particular, we discuss the difference in the $TE_{0,n}$ modes of rectangular waveguides. Although the partial differential equations for the longitudinal fields in the two waveguides are duals, the essential difference lies in the boundary conditions. The result is that in gyroelectric waveguides the general solution, in exact or approximate form, for the higher order modes is necessary. This result is unlike that for the gyromagnetic case in which the restricted $TE_{0,n}$ solutions are adequate for analysis of practical problems.

The following conventions are adopted: 1) all fields vary as exp $i(\kappa z-\omega t)$; 2) the magnetostatic field H_0 is along the x axis so that gyroelectric and gyromagnetic media are characterized, respectively, by the tensors

$$\boldsymbol{\varepsilon}=\epsilon\begin{bmatrix}1 & 0 & 0\\ 0 & \epsilon_1 & -i\epsilon_2\\ 0 & i\epsilon_2 & \epsilon_1\end{bmatrix} \quad (1)$$

$$\boldsymbol{\mu}=\mu\begin{bmatrix}1 & 0 & 0\\ 0 & \mu_1 & -i\mu_2\\ 0 & i\mu_2 & \mu_1\end{bmatrix}. \quad (2)$$

Semiconductors are included in gyroelectric media by viewing the conductivity tensor as part of an equivalent complex permittivity tensor; 3) modes are designated as $TE_{m,n}$ or $TM_{m,n}$. The first index refers to the field dependence on x, and the second index to y, irrespective of which coordinate is along the longer dimension of the guide. The dominant mode is $TE_{1,0}$ or $TE_{0,1}$, depending on whether the longer dimension is x_0 or y_0. The boundaries are at $x=(0, x_0)$ and $y=(0, y_0)$.

Consider the partial differential equations for E_z and H_z in gyroelectric waveguide,

$$\left[\frac{1}{k_0^2}\frac{\partial^2}{\partial x^2}+\frac{\epsilon_1}{k_1^2}\frac{\partial^2}{\partial y^2}+\epsilon_1-\frac{k^2\epsilon_2}{k_1^2}\right]E_z + \omega\mu\left[\kappa\frac{\epsilon_1-1}{k_1^2k_0^2}\frac{\partial}{\partial y}+\frac{\epsilon_2}{k_1^2}\right]\frac{\partial}{\partial x}H_z=0 \quad (3)$$

$$\omega\epsilon\left[\kappa\frac{\epsilon_1-1}{k_1^2k_0^2}\frac{\partial}{\partial y}-\frac{\epsilon_2}{k_1^2}\right]\frac{\partial}{\partial x}E_z + \left[\frac{1}{k_1^2}\frac{\partial^2}{\partial x^2}+\frac{1}{k_0^2}\frac{\partial^2}{\partial y^2}+1\right]H_z=0 \quad (4)$$

Manuscript received October 1, 1965; revised February 7, 1966.

where

$$k_0^2=k^2-\kappa^2$$
$$k_1^2=k^2\epsilon_1-\kappa^2; \qquad k^2=\omega^2\mu\epsilon.$$

These equations are exact duals of the equations for gyromagnetic waveguide derived by Vartanian and Jaynes [2]. The boundary conditions, however, are different in the two cases. Because of the special forms of the tensors (1) and (2), the boundary conditions of vanishing tangential E and normal B can be expressed in the alternative forms

$$E_z=0; \quad \frac{\partial H_z}{\partial x}=\frac{\partial H_z}{\partial y}=0 \quad (5)$$

for gyroelectric waveguide, and

$$E_z=0; \quad \frac{\partial H_z}{\partial x}=0; \quad \frac{\partial H_z}{\partial y}=-\kappa\frac{\mu_2}{\mu_1}H_z \quad (6)$$

for gyromagnetic waveguide. Thus, a solution of the boundary value problem for one type does not yield by duality the correct solution for the other, even though the differential equations are mathematically equivalent. The more significant difference appears in the $TE_{0,n}$ solutions.

The TE Solutions

It is well known that (3) and (4) do not admit TE or TM solutions unless the fields vary in one transverse coordinate only. In rectangular waveguides, this condition necessarily implies the TE modes, since E_z must vanish identically on the boundaries.

When the fields depend on y only, $\partial/\partial x=0$, (4) leads to the $TE_{0,n}$ solutions with wave numbers

$$\kappa^2=k^2-\left(\frac{n\pi}{y_0}\right)^2. \quad (7)$$

These wave numbers and the associated fields *do not* depend on the magnetostatic field. Hence, in gyroelectric waveguide, the $TE_{0,n}$ modes hold no interest. In contrast, in gyromagnetic waveguide, the dual of (3) yields the distorted $TE_{0,n}$ modes of Van Trier [1] with wave numbers

$$\kappa^2=k^2\frac{\mu_1^2-\mu_2^2}{\mu_1}-\left(\frac{n\pi}{y_0}\right)^2. \quad (8)$$

These modes depend on the external field and reduce to the conventional $TE_{0,n}$ modes when $H_0=0$.

When the fields depend on x only, $\partial/\partial y=0$, the equations for *both classes* of waveguides reduce to an inconsistent pair which admit only the trivial solution H_z = constant. Thus, the $TE_{m,0}$ modes do not exist.

In practice, the wave within the filled waveguide is launched from an empty section supporting the dominant $TE_{0,1}$ (or $TE_{1,0}$) mode, and the problem consists of determining the behavior of this mode as it passes through the media. The exact $TE_{0,n}$ solutions have adequately described gyromagnetic phenomena [3]. Thus, the general solution, with no restriction on the field variation, is seldom essential. In gyroelectric waveguide, however, if the wave is to depend on the magnetostatic field, all six components of the field are necessary, and these must depend on both transverse coordinates. The general solution, or approximations thereof, is then essential for analysis of gyroelectric phenomena. Moreover, since the $TE_{0,1}$ mode is independent of H_0, the magnetically controllable waves cannot be launched with this mode. If a dominant mode is used for the launching, it must be the $TE_{1,0}$.

Since the $TE_{1,0}$ mode is not admissible in the magnetized medium, it excites a mode which depends on both coordinates. Thus, a problem of the gyroelectric waveguide is to establish a quantitative connection between the $TE_{1,0}$ mode in the absence of H_0 and the form of the distorted wave in the presence of H_0. It will become evident that the task of finding the $TE_{1,0}$ limit of the general solution is a difficult one. A perturbation method [4], [5] was found to be simpler in establishing this connection. It is worth noting that the $TE_{0,n}$ limit of the general solution is not difficult to find because these modes are admissible restricted solutions.

The General Solution

We shall now give a brief account of a method for the rigorous general solution [6]. The solutions are of identical form in gyromagnetic and gyroelectric waveguides, but the conditional equations are not. Since a solution for the former has been given [2], we shall derive the conditional equation for the latter.

To separate variables, let

$$E_z=u(x)e(y) \quad (9a)$$
$$H_z=v(x)h(y), \quad (9b)$$

and substitute these in (3) and (4). After dividing the first equation by $v'e$ and the second by $u'h$, one obtains two equations of the form

$$G(x,y)=[f(x)+g(y)]\frac{u(x)}{v'(x)}+q(y)\equiv 0 \quad (10a)$$

$$F(x,y)=p(y)+[w(x)+r(y)]\frac{v(x)}{u'(x)}\equiv 0 \quad (10b)$$

where the primes denote differentiation. The key to separation of variables is that the variations of F and G with respect to x and y must vanish. This follows from the fact that F and G are themselves zero for all x and y. The vanishing variations constrain u and v to the relations

$$u'(x)=cv(x) \quad (11a)$$
$$v'(x)=-c'u(x) \quad (11b)$$

where c and c' are separation constants. With these relations we eliminate u and v from (3) and (4) to obtain the equations for e and h,

$$\left[a_1\frac{d^2}{dy^2}+b_1\right]e-\left[c_1\frac{d}{dy}+d_1\right]h=0 \quad (12a)$$

$$\left[c_2\frac{d}{dy}-d_2\right]e+\left[a_2\frac{d^2}{dy^2}+b_2\right]h=0 \quad (12b)$$

where the coefficients are defined by comparison to (3) and (4).

The separated equations allow rigorous solution for any set of boundary conditions, such as the set arising in a partially filled guide. For the completely filled waveguide, by (5) and (6), u and v' must vanish on $x=(0, x_0)$ in *both* types of waveguides. Thus the solutions for (11a) and (11b) are

$$u=\sin\alpha_m x, \qquad v=\cos\alpha_m x$$

with $\alpha_m=c=c'=m\pi/x_0$. The solutions of (12a) and (12b) are found to be combinations of trigonometric functions.

The conditional equation is obtained in the usual manner. After introducing the boundary conditions, one obtains two algebraic equations in two unknowns whose solution exists only if the determinant vanishes. In gyroelectric waveguide, the condition of vanishing e and h' on $y=(0, y_0)$ leads to the set of equations whose determinant is

$$\begin{aligned} P(\kappa, H_0) &= \frac{1}{(s_1^2-s_2^2)^2}\Big\{2\alpha_m^2\epsilon_2^2[\cos s_1y_0\cos s_2y_0-1] \\ &\quad -\frac{1}{s_1s_2}\left[\frac{\epsilon_1}{k^2}(k_1^2-\alpha_m^2)(s_1^2-s_2^2)^2 \right.\\ &\quad \left. -\alpha_m^2\epsilon_2^2(s_1^2+s_2^2)\right]\sin s_1y_0\sin s_2y_0\Big\} \end{aligned} \quad (13)$$

where

$$\begin{aligned} s_1^2+s_2^2 &= -2\kappa^2 - \frac{1}{\epsilon_1}[(\alpha_m^2-k^2\epsilon_1)(\epsilon_1+1)+k^2\epsilon_2^2] \\ s_1^2-s_2^2 &= \frac{1}{\epsilon_1}\{[(\alpha_m^2-k^2\epsilon_1)(\epsilon_1-1)+k^2\epsilon_2^2]^2 \\ &\quad +4\alpha_m^2k^2\epsilon_2^2\}^{1/2}. \end{aligned}$$

It is evident that evaluation of the roots of $P(\kappa, H_0)$ is a difficult problem. An attempt at expressing the roots κ as a Taylor series in powers of H_0 was not successful. In this series, the derivatives of κ were evaluated by implicit differentiation of $P(\kappa, H_0)$. Since the theory of implicit functions is valid for single-valued functions only, and since κ is a double-valued function of H_0, this method is not applicable. It is interesting that the second-order Taylor approximation gives the arithmetic mean of the two branches of κ calculated from degenerate perturbation theory. We also note that the limiting form of P for the $TE_{0,n}$ modes is obtained readily by setting $\alpha_m=0$, the value of α corresponding to no variation in x. The roots are then the correct ones given in (7). On the other hand, to find the roots for the wave to which the $TE_{m,0}$ mode is distorted, one needs to trace the variation of P as H_0 is changed from zero. This is a complicated process since every term in P is a function of H_0.

Conclusion

We have seen that, although gyromagnetic and gyroelectric waveguides are duals, their behaviors are different. The differences are particularly significant in the $TE_{0,n}$ modes. Gyroelectric phenomena in rectangular waveguide, unlike gyromagnetic phenomena, can be investigated only through the general solution which is too unwieldy for practical applications. Thus, approximation methods [5] are very desirable. We have also shown that the general solution can be obtained rigorously by the method of separation of variables.

G. J. Gabriel
Dept. of Elec. Engrg.
University of Notre Dame
Notre Dame, Ind.
M. E. Brodwin
Dept. of Elec. Engrg.
Northwestern University
Evanston, Ill.

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[6] G. J. Gabriel, Ph.D. dissertation, Northwestern University, Evanston, Ill., August 1964.

A High Directivity, Broadband Coaxial Coupler

Directional couplers having the proper impedance and directivity characteristics are useful devices for reflectometry measurements and for the intercomparison of RF power meters. In the latter application, stable, sensitive power detectors are connected to the appropriate coupler sidearms to allow monitoring of both the incident and reflected powers in the primary line.

While it is possible to achieve high directivity with ordinary couplers at a given frequency by use of auxiliary tuners, the tuning procedures are somewhat involved and time-consuming. It is thus more convenient to use couplers which are of inherent high directivity. In addition, tuning for high directivity may aggravate the impedance mismatch conditions at one or more of the coupler output ports.

It is the purpose of this correspondence to describe the design of a high (50-dB) directivity broadband (0.7 to 1.5 and 3 to 4 GHz) coaxial coupler having low VSWR (<1.01) in both the primary and secondary lines. Achievement of these characteristics was made possible to a large degree by the recent availability of precision coaxial connectors which are used on all ports. The basic design is more or less conventional in that two transmission lines are inductively and capacitively coupled by means of a common slot in their outer conductors. Inductive coupling is a function of the width of the slot, while capacitive coupling is a function of the spacing between center conductors. The coupling ratio K, as derived by Monteath,[1] is given by

$$K=\frac{1}{2}\left(k-\frac{1}{k}\right)\sin\beta l \qquad (1)$$

where βl is the length of the coupling slot in electrical wavelengths and k is a constant determined by the slot width and the spacing between center conductors. Thus, the coupling is a maximum at frequencies (f_c, $3f_c$, etc.) where l is an odd multiple of a quarter-wavelength, $\lambda/4$. The coupling is near zero at frequencies where l is an even multiple of $\lambda/4$. The bandwidth of the coupler, centered on each of the frequencies f_c, $3f_c$, etc., is roughly equal to f_c (i.e., 1150 MHz for the coupler described here).

High directivity was obtained primarily by designing the coupler so that the proper balance was obtained in the mutual capacitance and inductance between the coupled lines. Methods for calculating the proper spacing dimensions for a given coupling ratio are given by Monteath.[1] Of almost equal importance for broadband use is the elimination of impedance discontinuities within the coupler, at connectors, and in the load resistor which terminates the secondary line. Discontinuities cause reflections which, even though small, can significantly reduce the directivity. For example, in order to achieve 50-dB broadband directivity it is necessary that the reflection coefficient of internal discontinuities be reduced to the order of 0.002 or less. Usually, the largest discontinuity in coaxial couplers with coupling closer than about 30 dB occurs at the coupling slot. In this region the characteristic impedance Z_0 of both the primary and secondary lines tends to be lower than in the uncoupled region. In the 20-dB, 50-ohm coupler described here, the Z_0 in the coupled region as measured with a time domain reflectometer (TDR) was found to be 46 ohms. A wave propagating from the input would see an abrupt change in line impedance from 50 ohms to 46 ohms at the beginning of the coupling slot. The impedance remains 46 ohms along the length of the slot and then abruptly changes to 50 ohms again at the end. In order to bring the impedance in the coupled region back to 50 ohms, it was necessary to increase the effective outer to inner conductor diameter ratio. This was done by undercutting the outer conductors in the coupled region with a milling tool having a diameter (for convenience) that was the same as the original line diameter. Most of the undercutting was done in the lower and upper halves, respectively, of the primary and secondary outer conductors. Thus, in cross section, the shape of the outer conductors tended to become oblong in shape as shown in Fig. 1.

The coupler was made of brass with an outer conductor diameter of 0.5625 inch and

Manuscript received February 11, 1966, revised March 29, 1966.

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